Standard Model Expectations for CP Violation*

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Abstract

I review the predictions and expectations of the CKM model for CP violation in both the $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ systems. A brief discussion of CP violation in charged K- and B-decays is also included, as well as some remarks on the electric dipole moments of the neutron and the electron.

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I. Prologue

These notes contain a summary of the lectures I delivered at the School on Particle Physics and Cosmology held in Puri (Orissa), India, in January 1993. To keep the length of this manuscript manageable, I have not included here two topics which I discussed in Puri: the strong CP problem and invisible axions and CP violations and baryogenesis. The first topic is reviewed by me already rather comprehensively in Cecilia Jarlskog's monograph on CP violation [1] and it did not seem reasonable to repeat much of this discussion here again. The second topic, by itself, seemed somewhat disconnected from the rest of the other material and, regretfully, I decided to leave it out. For a discussion of some general issues involved in CP violation and baryogenesis, the interested reader is referred to my contribution to **25 Years of CP Violation** [2]. For a more thorough treatment of baryogenesis at the electroweak scale, the recent lectures of Shaposhnikov [3] are highly recommended, as is the rapporteur talk of A. Cohen at the PASCOS/Texas '92 Conference [4].

II. Summary of Experimental Information on CP Violation

I begin these lectures by reviewing what we know experimentally about CP violation. At present, we have only observed CP violation in the $K^0 - \bar{K}^0$ complex. However, important information on CP violation can also be deduced from the existing bounds on the electric dipole moment of the neutron and that of the electron, as well as from the ratio of baryons to photons in the universe.

II.A Measurement in the Neutral Kaon System

Christensen, Cronin, Fitch and Turlay's [5] observation of the decay $K_L \to \pi^+\pi^-$ occurred nearly 30 years ago. Since that time many sophisticated experiments have continued to probe for CP violation in the $K^0 - \bar{K}^0$ complex. At present, all positive signals of CP violation can be summarized in terms of 5 measured parameters: two complex amplitude ratios (η_{+-} and η_{00}) and the semileptonic rate differences (A_{K_L}). More precisely, the quantities measured

are:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv |\eta_{+-}| \; e^{i\phi_{+-}} \equiv \epsilon + \epsilon'$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv |\eta_{00}| \; e^{i\phi_{00}} \equiv \epsilon - 2\epsilon'$$

$$A_{K_L} = \frac{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) \; - \; \Gamma(K_L \to \pi^+\ell^-\bar\nu_\ell)}{\Gamma(K_L \to \pi^-\ell^+\nu_\ell) \; + \; \Gamma(K_L \to \pi^+\ell^-\bar\nu_\ell)} \tag{1}$$

To a very good approximation, one finds that

$$\eta_{+-} \; \simeq \; \eta_{00} \quad . \tag{2}$$

(and therefore $\epsilon' \ll \epsilon$) and that

$$A_{K_L} \; \simeq \; 2 \, Re\eta_{+-} \; ; \quad \phi_{+-} \; \simeq \; \phi_{00} \simeq \pi/4 \quad . \tag{3}$$

The first result above, as we shall see, tells one that CP violation in the neutral Kaon system is mostly due to mixing. In view of the fact that ϵ' is much less than ϵ, the latter two results provide tests of CPT conservation in neutral Kaon decays. Again, this will be elaborated upon below.

In more detail, there is at the moment conflicting evidence regarding ϵ'/ϵ, with the NA31 experiment at CERN reporting a 3σ positive signal for $Re \; \epsilon'/\epsilon$, but the E731 Fermilab experiment still finding a signal consistent with zero:

$$Re\frac{\epsilon'}{\epsilon} = \left\{ \begin{array}{ll} (23 \pm 7) \times 10^{-4} & [6] \\ (7.4 \pm 5.9) \times 10^{-4} & [7] \end{array} \right. \tag{4}$$

The imaginary part of this ratio, measured through the phase difference between ϕ_{+-} and ϕ_{00}, to the accuracy with which it is measured at present, is also consistent with zero. One finds

$$3 \, Im \; \epsilon'/\epsilon \; = \phi_{+-} - \phi_{00} = \left\{ \begin{array}{ll} (-0.2 \pm 2.6 \pm 1.2)^0 & [8] \\ (1.6 \pm 1.0 \pm 0.7)^0 & [7] \end{array} \right. \tag{5}$$

In addition to the above, the compilation of the Particle Data Group [9], gives

$$|\eta_{+-}| \; = (2.268 \pm 0.023) \times 10^{-3} \tag{6a}$$

$$A_{K_L} \; = (3.27 \pm 0.12) \times 10^{-3} \tag{6b}$$

and

$$\phi_{+-} = (46 \pm 1.2)^0 \ . \tag{6c}$$

However, this last number has been brought into question by the recent reanalysis performed by the E731 collaboration. Using a somewhat smaller value of the $K_L - K_S$ mass difference Δm, the E731 reanalysis yields, instead of the PDG value above [9], the average value [7]

$$\phi_{+-} = (42.8 \pm 1.1)^0 \tag{7}$$

This value is in much better accord with what one predicts from CPT conservation [10] where one expects[1]

$$\phi_{+-} \simeq \phi_{sw} = \tan^{-1} \frac{2\Delta m}{\Gamma_S - \Gamma_L} = (43.4 \pm 0.2)^0 \tag{8}$$

II.B Bounds on Electric Dipole Moments

Landau [11] was the first to point out that, for an elementary particle, having an electric dipole moment violates both P and T. If one assumes that CPT is conserved, as is expected from the CPT theorem [12], then the presence of an electric dipole moment would also signal CP violation.

A simple argument to see why an electric dipole moment $\vec{d}$ violates both P and T is as follows [13]. Since $\vec{d}$ is a 3-vector, and measures a static property of an elementary particle, it must be proportional to the only other 3-vector in the problem - the angular momentum $\vec{J}$. Thus

$$\vec{d} = d\vec{J} \ . \tag{9}$$

However, $\vec{d}$ is odd under P, while $\vec{J} \to \vec{J}$ under parity. Hence, if P is conserved, the constant d must vanish. Similarly, $\vec{d}$ is even under T, but $\vec{J} \to -\vec{J}$ under time reversal transformations. Hence again, unless T is violated, d must vanish.

Experimentally, there are strong limits on the electric dipole moments of both the neutron and the electron. From the Particle Data Group [9] one has

$$\begin{aligned} d_n &< 1.2 \times 10^{-25} \text{ecm} \quad (95\% \ C.L.) \\ d_e &= (-0.3 \pm 0.8) \times 10^{-26} \text{ecm} \end{aligned} \tag{10}$$

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[1]Here Γ_S and Γ_L are the widths of K_S and K_L, respectively.

II.C Astrophysical and Cosmological Information on CP Violation

There is a nice cosmological argument, due to Zeldovich, Kobzarev and Okun [14] which strongly suggests that the violation of CP seen experimentally in the neutral Kaon system must signal an **explicit** violation of this symmetry in the Lagrangian of the theory, rather than a spontaneous breaking of CP by the vacuum. If CP were to be a spontaneously broken symmetry, with a breaking scale $v_{\not{C}P}$, one would expect that at temperatures below $T^* \sim v_{\not{C}P}$ different CP domains would form in the universe. These domains would be separated from each other by domain walls of typical surface energy density

$$\sigma \sim T^{*3} \sim v_{\not{C}P}^3 \quad . \tag{11}$$

However, unless $v_{\not{C}P}$ is extremely small, which is not sensible since one expects that $v_{\not{C}P}$ be at least as big as the scale of electroweak breaking $v \sim 250 \; GeV$, the energy in these domain walls today would far exceed the closure density of the universe. [2] One has

$$\rho_{\text{wall}} \sim \sigma T \sim v_{\not{C}P}^3 T \sim 10^{-7} \left(\frac{v_{\not{C}P}}{TeV} \right)^3 \; GeV^{-4}, \tag{12}$$

to be compared to the closure density of the universe today

$$\rho_{\text{closure}} \sim 10^{-46} \; GeV^{-4} \quad . \tag{13}$$

There is a second place where cosmology and astrophysics have a bearing on the issue of CP violation, related to the ratio η of baryons to photons in the universe today. This ratio is rather well determined from the study of the primordial abundances of the light elements produced in nucleosynthesis and one finds[15]

$$3.7 \times 10^{-10} < \eta < 4.0 \times 10^{-10} \quad . \tag{14}$$

If the universe was symmetric in the number of baryons and antibaryons at temperatures above a few GeV, then from subsequent annihilations $(p + \bar{p} \rightarrow$

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[2]This problem can be avoided in inflationary universe scenarios if $v_{\not{C}P}$ is greater than the scale when inflation takes place. However, then one has to worry about transmitting spontaneous CP violation occuring at these very large scales to the neutral Kaon sector [2].

2γ) one would expect a ratio η only of order $\eta \sim 10^{-18}$[16]. Therefore, this ratio must reflect a primordial baryon-antibaryon asymmetry. That is,

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \quad . \tag{15}$$

It is possible - but very difficult to conceive physically - that η is an initial condition for our universe. If so, one learns nothing from this number. However, it is much more reasonable that η be produced dynamically in the course of the evolution of the universe. In this case, as Sakharov [17] was the first to point out, the dynamics necessarily must involve CP violating phenomena. Thus the ratio η itself is also a measure of CP violation. The relation of η with the CP violating phenomena seen in the kaon system is, however, far from direct [2], even for baryogenesis produced at the electroweak scale. [3]

III.   CKM Paradigm

There is an important consequence that follows from assuming that the observed CP violation is due to explicit CP breaking in the underlying Lagrangian. Namely, if we want this Lagrangian to be renormalizable, then once CP is no longer a symmetry it follows that **all** parameters of this Lagrangian that can be complex must be so. Otherwise, one could not absorb potential infinities into appropriately complex counter terms. In the standard model of the strong and electroweak interactions, the gauge sector is necessarily real so no CP phases can enter through the gauge coupling constants.[3]. It follows, therefore, that any CP violation in this model must arise as a result of interactions in the Higgs sector.

If one has only one complex doublet of Higgs fields Φ, as is generally assumed in the simplest version of the standard electroweak theory, then any CP violating phases can only appear in the Yukawa interactions because, by Hermiticity, the Higgs potential has only real parameters:

$$V = \lambda(\Phi^\dagger\Phi - \frac{v^2}{2})^2 \quad , \tag{16}$$

[3]There can be CP violation associated with the gauge interactions as a result of the presence of non-trivial vacuum angles θ. This matter is not very germane to the present discussion, and will not be examined here further. For a discussion, see, for example[1].

with λ and v^2 real. However, when one has more than one Higgs doublet, CP violating phases can also enter in the pure Higgs sector. For instance, with two Higgs doublets, Φ_1, and Φ_2, one can have a complex mass term

$$\mathcal{L} = -\mu^2 \Phi_1 \Phi_2 - (\mu^2)^* \Phi_1^\dagger \Phi_2^\dagger \ . \tag{17}$$

It is often useful in confronting novel phenomena to describe them in a model with the minimum number of free parameters. This is precisely what occurs with CP violation in the standard electroweak model with only one Higgs doublet, in the case in which there are three generations of quarks (and leptons). In this physically relevant circumstance, all CP violating phenomena are traceable to a single phase arising from the Yukawa couplings of quarks to the doublet Higgs boson. This is the well known Cabibbo Kobayashi Maskawa (CKM) paradigm [18]. I shall, in what follows, on the whole, concentrate on the prediction of this model. There may well be further CP violating phases in nature besides the CKM phase. However, since we know that the CKM phase must exist by the renormalizability of the standard model, it seems reasonable to see first if this phase indeed can explain all the observed CP violating phenomena in the Kaon system.

III.A   Counting CP phases in the Standard Model

If CP is not a good symmetry, the Yukawa couplings of quarks to the Higgs doublet Φ and its complex conjugate $\tilde{\Phi} = i\tau_2 \Phi^*$ are necessarily complex. If Q^i, u_R^i and d_R^i denote, respectively, the left-handed quark doublet of the ith generation and the charge 2/3 and charge $-1/3$ right-handed quarks of this same generation, one can write these Yukawa interactions as

$$\mathcal{L}_{\text{Yukawa}} = \Gamma_{ij}^u \bar{Q}_L^i u_R^j \Phi + \Gamma_{ij}^d \bar{Q}_L^i d_R^j \tilde{\Phi} + h.c. \tag{18}$$

When Φ and $\tilde{\Phi}$ are replaced by their vacuum expectation values, the above interactions will give rise to complex mass matrices for the quarks. To go to a **physical basis** where the quarks have real diagonal masses, one must perform a unitary transformation on the quark fields which, in general, will involve different unitary matrices for the left-handed and right-handed fields and different matrices for the charge 2/3 and charge $-1/3$ fields:

$$u_{L,R} = U_{L,R}^u u_{L,R} \quad ; \quad d_{L,R} = U_{L,R}^d d_{L,R} \ . \tag{19}$$

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As a result of this basis change, the interaction of the gauge fields with the quarks, which used to be family diagonal, now no longer are necessarily so. Neutral current interactions, since they involve always $(U_{L,R}^u)^\dagger U_{L,R}^u = 1$ or $(U_{L,R}^d)^\dagger U_{L,R}^d = 1$, continue to be diagonal. However, for charged current interactions what enters after the basis change is the unitary matrix

$$V_{CKM} = (U_L^u)^\dagger U_L^d \quad, \tag{20}$$

or its adjoint. This gives rise to family mixing .

In this physical basis all the CP violating phase information present in the Yukawa couplings is transferred to the Cabibbo Kobayashi Maskawa matrix V_{CKM}. Because this matrix is unitary, for N_g generations of quarks V_{CKM} is parameterized by $\frac{1}{2}N_g(N_g-1)$ real angles and $\frac{1}{2}N_g(N_g+1)$ phases. However, not all of these phases are physical since one can absorb $(2N_g - 1)$ phases by appropriate quark field redefinitions [4]. Thus in V_{CKM} there are in total $\frac{1}{2}(N_g-1)(N_g-2)$ physical phases. As a result, as I alluded to earlier, in the physically relevant case of 3 generations of quarks there is only one physical phase δ in V_{CKM}, which is responsible for all CP violating phenomena. This is the CKM paradigm [5].

III.B   The Wolfenstein Parameterization of the CKM Matrix

The 3×3 unitary matrix V_{CKM} characterizing charged current weak interactions can be specified in many equivalent forms. It proves convenient to adopt a standard parametrization [9] which admits a simple and useful approximate form [19]. One writes

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

[4]An overall phase cannot be redefined away if CP is not conserved.

[5]In principle, there is an analogous matrix to V_{CKM} in the leptonic sector of the standard model. However, if the neutrinos are massless, one can absorb this matrix entirely by redefining once more the neutrino fields, since they are mass degenerate. Because CP violation in the lepton sector is connected with neutrino mass generation, I shall not discuss it further here. I note only that, for analogous reasons, CP violating effects in the quark sector in the CKM paradigm will vanish in the limit when the quarks become mass degenerate.

$$
= \begin{bmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{bmatrix} , \quad (21)
$$

where

$$
c_i \equiv \cos\theta_i \ ; \ \ s_i \equiv \sin\theta_i \ . \tag{22}
$$

The above is well approximated to $0(\lambda^4)$ by writing for the s_i the hierarchical parametrization [19]

$$
\sin\theta_1 = \lambda \ ; \ \ \sin\theta_2 = A\lambda^2 \ ; \ \ \sin\theta_3 = A\sigma\lambda^3 \ . \tag{23}
$$

Here λ is essentially the sine of the Cabibbo angle and one has, experimentally,

$$
\lambda \simeq \sin\Theta_c = 0.22 \ , \tag{24}
$$

while A and σ, as we shall see, turn out to be of $0(1)$. In terms of the above, (Wolfenstein) parametrization one can write V_{CKM} to $0(\lambda^4)$ as

$$
V_{CKM} = \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 \sigma e^{-i\delta} \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \sigma e^{i\delta}) & -A\lambda^2 & 1 \end{bmatrix} , \tag{25}
$$

or, more conventionally, writing $\sigma e^{-i\delta} = \rho - i\eta$

$$
V_{CKM} = \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} . \tag{26}
$$

I note for future use that to $0(\lambda^4)$ the CKM matrix has only two elements which have an imaginary part, V_{ub} and V_{td}. Furthermore, from the form of the matrix one sees that information on the, still to be determined, parameters A and σ (or $\sqrt{\rho^2 + \eta^2}$) necessitates measurements involving b quarks, with A being fixed by V_{cb} and σ (or $\sqrt{\rho^2 + \eta^2}$) by the ratio of $|V_{ub}|/|V_{cb}|$. Obviously, information on the phase δ (or the CP violating parameter η) can be gotten from the measurements of CP violating phenomena in the $K^0 - \bar{K}^0$ complex. However, because δ (and η) also enter in V_{td} some useful information on this parameter can also be garnered from non CP violating phenomena, like $B_d^0 - \bar{B}_d^0$ mixing which depend on this matrix element. We will return to discuss how well A, σ and δ (or A, ρ and η) are determined at present experimentally, after we discuss the predictions of the CKM paradigm for CP violation in the Kaon system.

IV. CP Violation in the Kaon System

To compare the experimental values of the various CP violating parameters which are measured in the $K^0 - \bar{K}^0$ complex and which we discussed in Section II, it is necessary to develop a bit of formalism. This formalism will also be relevant later on when I will discuss CP violation in the B system.

IV.A   Two State Formalism

Neutral particle - antiparticle systems ($P - \bar{P}$ systems), like those formed by a $K^0 \sim d\bar{s}$ and a $\bar{K}^0 \sim \bar{d}s$ or by a $B_d^0 \sim d\bar{b}$ and a $\bar{B}_d^0 \sim \bar{d}b$ meson, provide very nice examples of quantum mechanics at work. The individual states in these systems, P and $\bar{P}$, are unstable due to the weak interactions ($\Delta P = \pm 1$ processes). Furthermore, P and $\bar{P}$ can mix with each other via a 2nd order weak process ($\Delta P = \pm 2$ processes). It is useful to describe the decay and the mixing in the $P - \bar{P}$ complex by means of an effective 2×2 Hamiltonian [20]

$$H_{\text{eff}} = M - \frac{i}{2}\Gamma \ , \tag{27}$$

characterized by Hermitean mass, M, and decay, Γ, matrices. The time evolution of the system is then described by the 2-state Schroedinger equation:

$$i\frac{\partial}{\partial t}\left(\begin{array}{c} P \\ \bar{P} \end{array}\right) = H_{\text{eff}} \ \left(\begin{array}{c} P \\ \bar{P} \end{array}\right) \ . \tag{28}$$

If CPT is conserved, as I shall assume in what follows, then the matrices M and Γ have a further constraint on them besides their Hermiticity, namely

$$M_{11} = M_{22} \ \ ; \ \ \Gamma_{11} = \Gamma_{22} \ . \tag{29}$$

This constraint just reflects the simple fact that CPT requires particles and antiparticles to have the same mass and the same lifetime. If in addition CP were to be a good symmetry - in the phase convention where $CP|P> = |\bar{P}>$ - one would have the further restriction that

$$M_{12} = M_{12}^* \ \ ; \ \ \Gamma_{12} = \Gamma_{12}^* \ . \tag{30}$$

That is, the matrices M and Γ would be real. Obviously, in the CKM paradigm this will not be so because of the presence of the phase δ.

It is straightforward to deduce the physical eigenstates of H_{eff}. These are the states $|P_\pm>$ which have the simple time evolution

$$|P_\pm(t)>= e^{-im_\pm t}e^{-\frac{1}{2}\Gamma_\pm t}|P_\pm> \quad . \tag{31}$$

That is, they are definite mass eigenstates, $m_\pm$, which decay with fixed rates $\Gamma_\pm$. Assuming CPT conservation, but not assuming CP conservation, one finds that the states $|P_\pm>$ are the following combinations of $|P>$ and $|\bar P>$ states

$$|P_\pm>= \frac{1}{\sqrt{2(1+|\epsilon_P|^2)}}\left\{(1+\epsilon_P)|P> \pm (1-\epsilon_P)|\bar P>\right\} \quad . \tag{32}$$

Here the complex parameter ϵ_P characterizes the amount of CP violation in the evolution of the system. Obviously, if ϵ_P where to vanish then the physical states $|P_\pm>$ would be CP eigenstates. One finds that

$$\frac{1-\epsilon_P}{1+\epsilon_P} = \frac{[M_{12}^* - \frac{i}{2}\Gamma_{12}^*]^{1/2}}{[M_{12} - \frac{i}{2}\Gamma_{12}]^{1/2}} \equiv \eta_P e^{i\Phi_p} \quad . \tag{33}$$

Because the states $|P_\pm>$ are the ones that have a definite time evolution, and these are superpositions of $|P>$ and $|\bar P>$, it follows that if one produces at $t=0$ a state $|P>$ this state will evolve in time into a superposition of $|P>$ and $|\bar P>$ states. A simple calculation gives the following formula for the resulting state - which I'll call $|P_{\text{phys}}(t)>$ - at time t:

$$|P_{\text{phys}}(t)>= e^{-\frac{i}{2}(m_++m_-)t}e^{-\frac{1}{4}(\Gamma_++\Gamma_-)t}\{a(t)|P> +b(t)|\bar P>\} \tag{34}$$

where
$$a(t) = \cos\frac{\Delta Ht}{2} \quad ; \quad b(t) = i\eta_P e^{i\Phi_p}\sin\frac{\Delta Ht}{2} \quad . \tag{35}$$

Here the parameter ΔH contains information of the physical parameters of the $P - \bar P$ complex

$$\Delta H = (m_+ - m_-) - \frac{i}{2}(\Gamma_+ - \Gamma_-) \quad . \tag{36}$$

The parameter $b(t)$ above contains information on CP violation in the system. However, in general, this will not be the only place where CP violating effects can enter.

Besides CP violation arising through the evolution and mixing of P and $\bar{P}$, there can be CP violating phases which enter directly in the decay amplitudes of P and $\bar{P}$ to some final states f. In the CKM paradigm, since V_{CKM} enters precisely in $\Delta P = \pm 1$ decays, one expects to have non trivial CP violating phases in the decay amplitudes $A(P \to f)$. Thus observable effects of CP violation in the $P - \bar{P}$ complex are generally mixtures of decay ($\Delta P = \pm 1$) and mixing ($\Delta P = \pm 2$) CP violating parameters. Although in the CKM model all these parameters are related to the phase δ, it is difficult in general to relate directly CP violating observables to the underlying theory. Nevertheless, we will see that the CKM model has considerably different predictions for CP violation in the Kaon complex than it does in the D^0, B_d^0 and B_s^0 systems. Kaons are a special case since $\Gamma_+ \gg \Gamma_-$ and already specific decays, like $K_L \to \pi^+\pi^-$, are direct signals of CP violation. In contrast, for the most part in Kaon CP violation one is only able to give qualitative tests of the theory. On the other hand, in the B_d^0 and B_s^0 systems, as we shall see, one can disentangle better some of the dynamical complications which ensue in trying to compare theory with experiment. As a result, one can hope that future measurements of CP violation in these systems may really provide quantitative tests of the theory.

IV.B Neutral Kaon Amplitudes: ϵ and ϵ'

The parameters ϵ and ϵ', which we defined in Sec. II, can be related in a straightforward manner to the K decay amplitudes and the mixing parameters ϵ_K. Working to lowest order in small quantities one has

$$
\begin{aligned}
|K_S> &\simeq \tfrac{1}{\sqrt{2}}\{(1+\epsilon_K)|K^0> +(1-\epsilon_K)|\bar{K}^0>\} \\
|K_L> &\simeq \tfrac{1}{\sqrt{2}}\{(1+\epsilon_K)|K^0> -(1-\epsilon_K)|\bar{K}^0>\} \ .
\end{aligned}
\tag{37}
$$

Let me denote the amplitudes for a K^0 to decay into a $\pi\pi$ state of isospin I by

$$
< 2\pi; I|T|K^0 >= A_I e^{i\delta_I} \ , \tag{38}
$$

where δ_I is the $\pi - \pi$ scattering phase shift in the channel of isospin I. Then the corresponding amplitude for $\bar{K}^0$ decay is

$$
< 2\pi; I|T|\bar{K}^0 >= A_I^* e^{i\delta_I} \ . \tag{39}
$$

That is, this amplitude will have the same strong scattering phase factor but, if there is $\Delta S = 1$ CP violation, this amplitude will differ from that for K^0 decay since it involves A_I^*. This is what one expects in the CKM paradigm.

Using the isospin decomposition

$$
\begin{aligned}
|\pi^+\pi^-> &= \sqrt{\frac{2}{3}} \mid 2\pi; 0 > + \sqrt{\frac{1}{3}} \mid 2\pi; 2 > \\
|\pi^0\pi^0> &= \sqrt{\frac{1}{3}} \mid 2\pi; 0 > - \sqrt{\frac{2}{3}} \mid 2\pi; 2 > \quad ,
\end{aligned}
\tag{40}
$$

and expanding in small quantities again, it is easy to derive formulas for

$$
\eta_{+-} = \frac{<\pi^+\pi^-|T|K_L>}{<\pi^+\pi^-|T|K_S>} \quad ; \quad \eta_{00} = \frac{<\pi^0\pi^0|T|K_L>}{<\pi^0\pi^0|T|K_S>}
\tag{41}
$$

and whence formulas for ϵ and ϵ'. Because experimentally $|A_2|/|A_0|$ is small, of $0(1/20)$, it suffices also to retain only terms in first order in this quantity - the so called $\Delta I = 3/2$ suppression. One finds

$$
\begin{aligned}
\epsilon &\simeq \epsilon_K + i\frac{ImA_0}{ReA_0} \\
\epsilon' &\simeq \frac{i}{\sqrt{2}}e^{i(\delta_2 - \delta_0)}\frac{ReA_2}{ReA_0}\left[\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}\right]
\end{aligned}
\tag{42}
$$

The parameter ϵ' which measures the difference between η_{+-} and η_{00} is obviously suppressed by the small factor of ReA_2/ReA_0, but this by itself cannot account for the very small value of ϵ'/ϵ seen experimentally.

In the older literature, one often used a phase convention suggested by Wu and Yang [21] to simplify the formulas for ϵ and ϵ'. Wu and Yang made use of the freedom of choosing a CP phase for how $|K^0>$ and $|\bar{K}^0>$ are related under CP,

$$
CP|K^0 >= e^{i\xi}|\bar{K}^0 > \quad ,
\tag{43}
$$

and chose ξ so as to make $ImA_0 = 0$. Then $\epsilon \equiv \epsilon_K$. I find it more physical to use the **quark phase convention** where $\xi = 0$ and the phases for A_0 and A_2 just follow from the CKM phase that enters at the quark vertices. Nevertheless, even in this case, one can still arrive at some simplifications

by using the fact that the 2π intermediate state is by far the dominant contribution in the width matrix Γ.

Using the formulas given previously, it is easy to show that the mixing parameter ϵ is given by the equation

$$\epsilon_K = \left[\frac{-ImM_{12} + \frac{i}{2}Im\Gamma_{12}}{i\Delta H}\right]_K \quad . \tag{44}$$

The kinematical parameter $(\Delta H)_K$ is related to the $K_L - K_S$ mass difference Δm and the superweak phase ϕ_{sw}

$$i(\Delta H)_K = \frac{1}{2}(\Gamma_S - \Gamma_L) - i(m_L - m_S) \simeq \sqrt{2}\Delta m e^{-i\phi_{sw}} \quad . \tag{45}$$

Because of the 2π dominance in the K^0 amplitudes one has that

$$\Gamma_S - \Gamma_L \simeq \Gamma_S \sim 2(ReA_0)^2 \quad , \tag{46}$$

while

$$\Gamma_{12} \sim (A_0^*)^2 \quad . \tag{47}$$

Thus

$$\frac{Im\ \Gamma_{12}}{\Gamma_S - \Gamma_L} \simeq -\frac{ImA_0}{ReA_0} \quad . \tag{48}$$

Using this result, along with the definition of ϵ_K and the fact that $\Delta m \simeq \frac{1}{2}(\Gamma_S - \Gamma_L)$, one easily deduces that [22]

$$\epsilon = \epsilon_K + i\frac{ImA_0}{ReA_0} \simeq \frac{1}{\sqrt{2}}e^{i\phi_{sw}}\left[-\frac{ImM_{12}}{\Delta m} + \frac{ImA_0}{ReA_0}\right] \quad . \tag{49}$$

Several remarks are in order:

i.) From the above formula for ϵ one sees that one expects that the phase of ϵ should be the superweak phase ϕ_{sw}. Since $\epsilon' \ll \epsilon$, the phase of ϵ is just that of η_{+-} and indeed experimentally $\phi_{+-} \simeq \phi_{sw}$. This result is actually a CPT test, for one can show that the inclusion of CPT violating effects, both in the evolution of the $K^0 - \bar{K}^0$ system and in the decay amplitudes, produces contributions to ϵ with a phase $\phi_{sw} + \pi/2$ [22][23].

ii.) Since there are no CP violating phases in the semileptonic decay amplitudes of K^0 and $\bar{K}^0$, the semileptonic asymmetry A_{K_L} can only depend on the mixing CP violating parameter ϵ_K. A simple calculation then secures the formula

$$A_{K_L} = 2Re\ \epsilon_K \ .\tag{50}$$

Since $Re\ \epsilon_K = Re\ \epsilon \simeq Re\ \eta_{+-}$, one sees that the observed relationship $A_{K_L} \simeq 2Re\ \eta_{+-}$ follows directly. Again, the extent by which $Re\ \epsilon - \frac{1}{2}A_{K_L}$ differes from zero is a test of CPT, for if CPT were not conserved both the formula for ϵ and A_{K_L} would contain further terms. Using the experimental values for A_{K_L}, η_{+-} and the phase ϕ_{+-} one finds [10]

$$Re\ \epsilon - \frac{1}{2}A_{K_L} = \left\{ \begin{array}{ll} (-0.6 \pm 0.7) \times 10^{-4} & [9] \\ (0.3 \pm 0.7) \times 10^{-4} & [7] \end{array} \right. ,\tag{51}$$

where the two different values above result from using, respectively, the PDG value for ϕ_{+-} and that of E731.

iii.) The values for the $\pi\pi$ phase shift difference [24], $\delta_2 - \delta_0 = -(45 \pm 6)^0$, is such that the phase of ϵ' is also very near 45^0. Thus to a very good approximation

$$Re\frac{\epsilon'}{\epsilon} \simeq \frac{\epsilon'}{\epsilon} \simeq \left[\frac{ReA_2}{ReA_0}\right] \left[\frac{\frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0}}{-\frac{ImM_{12}}{\Delta m} + \frac{ImA_0}{ReA_0}} \right] \ .\tag{52}$$

Experimentally ϵ'/ϵ is much below the $\Delta I = 3/2$ suppression factor of $ReA_2/ReA_0 \sim 1/20$. Hence, barring an accidental cancellation in the numerator one deduces that

$$|\frac{ImA_0}{ReA_0}| \ll |\frac{ImM_{12}}{\Delta m}|\tag{53}$$

and hence that $|\epsilon|$ should be well approximated by the simple formula

$$|\epsilon| \simeq \frac{|ImM_{12}|}{\sqrt{2}\Delta m} \ .\tag{54}$$

That is, $|\epsilon|$ is essentially only a result of mixing CP violation.

Figure 1: Box diagram giving rise to the mass mixing matrix element M_{12}.

IV.C   Boxes and Penguins - Standard Model Predictions.

In the standard electroweak model M_{12} arises from the 2nd order weak box diagram shown in Fig. 1. This diagram gives rise to an effective $\Delta S = 2$ Lagrangian

$$\mathcal{L}_{\Delta S=2} = A_{\text{box}}[\bar{d}(\gamma_\mu(1-\gamma_5)s)(\bar{d}\gamma^\mu(1-\gamma_5)s)] + h.c., \tag{55}$$

where the coefficient A_{box} is easily read out from Fig. 1. One has

$$
\begin{aligned}
A_{\text{box}} \;=\; & \int \frac{d^4q}{(2\pi)^4} D^W_{\mu\alpha}(q) D^W_{\nu\beta}(q) \;\;. \\
& \cdot \sum_{ij} \lambda_i[\gamma^\mu(1-\gamma_5)D_i(q)\gamma^\nu(1-\gamma_5)] \\
& \lambda_j[\gamma^\beta(1-\gamma_5)D_j(q)\gamma^\alpha(1-\gamma_5)]
\end{aligned}
\tag{56}
$$

Here $D^W_{\mu\alpha}(q)$ and $D_i(q)$ are the W and i^{th} fermion propagators, respectively, while the coefficients λ_i involve the following products of CKM matrix elements:

$$\lambda_i = V_{is}V_{id}^* \;\;. \tag{57}$$

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The integral in A_{box} is potentially quadratically divergent. However, the unitarity of the CKM matrix implies that

$$\lambda_u + \lambda_c + \lambda_t = 0 \quad . \tag{58}$$

Eliminating λ_u in the expression for A_{box} gives differences of terms and these lead to a convergent expression for A_{box}. This is the celebrated GIM mechanism [25]. A straightforward calculation [26] secures the following formula for A_{box}.

$$A_{\text{box}} = \frac{G_F^2}{16\pi^2}\{\lambda_c^2 m_c^2 \eta_1 + \lambda_t^2 m_t^2 f_2(y_t)\eta_2 + 2\lambda_c\lambda_t m_c^2(\ell n \frac{m_t^2}{m_c^2} + f_3(y_t))\eta_3\} \quad . \tag{59}$$

Here $f_2(y_t)$ and $f_3(y_t)$ are kinematical functions which are weakly dependent on the ratio of the top mass to the W-mass, $y_t = m_t^2/M_W^2$. One finds [26]

$$
\begin{aligned}
f_2(y_t) &= 1 - \frac{3y_t(1+y_t)}{4(1-y_t)^2}\left[1 + \frac{2y_t}{1-y_t^2}\ell n y_t\right]\\
f_3(y_t) &= \frac{-3y_t}{4(1-y_t)}\left[1 + \frac{y_t}{1-y_t}\ell n y_t\right] \quad .
\end{aligned}
\tag{60}
$$

The coefficient η_i are QCD short distance corrections to the box graph of Fig. 1, arising from festooning this graph with gluons. These coefficients have been calculated by Gilman and Wise [27] and they find, approximately,

$$\eta_1 \simeq 0.7 \quad ; \quad \eta_2 \simeq 0.6 \quad ; \quad \eta_3 \simeq 0.4 \quad . \tag{61}$$

To be more precise, the actual values for the η_i are dependent on the scale μ at which the operator entering in $\mathcal{L}_{\Delta S=2}$ is normalized. However, the matrix element of this operator also depends on μ, with physical cogency requiring that the product of the η_i and the matrix element be $\mu-$ independent. It has become conventional to write this matrix element in terms of a constant $B_K(\mu)$, which is normalized so that the value $B_K = 1$ corresponds to the vacuum insertion approximation to this matrix element. That is

$$< K^0|\bar{d}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma^\mu(1-\gamma_5)s|\bar{K}^0> = \frac{8}{3}f_K^2 M_K^2 B_K(\mu^2) \quad , \tag{62}$$

with $f_K \simeq 160 \ MeV$ being the Kaon decay constant. The best evaluation of $B_K(\mu)$ obtained in lattice QCD, for the μ values which give the η_i values given above, is [28]

$$B_K(\mu) = 0.80 \pm 0.20 \quad . \tag{63}$$

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For the computation of ϵ one needs to know ImM_{12} and one has

$$ImM_{12} = \frac{Im < K^0|\mathcal{L}_{\Delta S=2}|\bar{K}^0 >}{2M_K} = \frac{4}{3}f_K^2 M_K B_K ImA_{\text{box}} \quad . \qquad (64)$$

A_{box} only contains an imaginary part as a result of the nontrivial phase δ in the CKM matrix. To the accuracy one is working here, it does not suffice to approximate V_{cd} to $0(\lambda^3)$ but one must retain its contributions to $0(\lambda^5)$, where one has

$$V_{cd} = -\lambda[1 + A^2\sigma\lambda^4 e^{+i\delta}] \quad . \qquad (65)$$

To leading order in λ in both the real and imaginary parts one finds

$$
\begin{aligned}
\lambda_c^2 &= (V_{cs}V_{cd}^*)^2 \simeq \lambda^2 - 2iA^2\sigma\lambda^6\sin\delta \\
\lambda_t^2 &= (V_{ts}V_{td}^*)^2 \simeq A^4\lambda^{10}\left\{[(1-\sigma\cos\delta)^2 - \sigma^2\sin^2\delta] + 2i\sigma(1-\sigma\cos\delta)\sin\delta\right\} \\
2\lambda_c\lambda_t &= 2(V_{cs}V_{cd}^*V_{ts}V_{td}^*) = 2A^2\lambda^6\left\{(1-\sigma\cos\delta) + i\sigma\sin\delta\right\} \quad .
\end{aligned}
\qquad (66)
$$

Even though $m_t^2 \gg m_c^2$, for the real part of A_{box} the only relevant piece is that proportional to λ_c^2 and $\lambda_c^2 \simeq \lambda^2$. However, for ImA_{box} the contributions of the λ_c^2 and $2\lambda_c\lambda_t$ terms are comparable, being both proportional to λ^6. Furthermore, even though $Im\lambda_t^2 \sim \lambda^{10}$, the multiplying factor of m_t^2 rather than m_c^2 in A_{box} does not allow one to neglect this term.

Collecting all this information together, one arrives at the following master formula for ϵ in the standard model [29]:

$$
\begin{aligned}
|\epsilon| \simeq \frac{|ImM_{12}|}{\sqrt{2}\Delta m} \quad &\simeq \quad \frac{G_F^2 f_K^2 M_K}{6\sqrt{2}\pi^2\Delta m}B_K[A^2\sigma\lambda^6\sin\delta] \quad . \\
&\cdot\left\{m_c^2[-\eta_1 + \eta_3\left(\ell n\frac{m_t^2}{m_c^2} + f_3(y_t)\right)] + \right. \\
&\left. + m_t^2\eta_2 f_2(y_t)\left[A^2\lambda^4(1-\sigma\cos\delta)\right]\right\} \quad . \qquad (67)
\end{aligned}
$$

I will discuss shortly a more detailed comparion of this formula with experiment. However, it is useful to get first an order of magnitude estimate of the expected size of ϵ. The $K_S - K_L$ mass difference Δm, neglecting possible long distance contributions[30], is given by

$$\Delta m \simeq Re\frac{< K^0|\mathcal{L}_{\Delta S=2}|\bar{K}^0 >}{2m_K} \quad . \qquad (68)$$

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Thus a rough estimate of ϵ is provided by

$$|\epsilon| \simeq \frac{ImA_{\text{box}}}{ReA_{\text{box}}} \simeq A^2\sigma\lambda^4\sin\delta\Big\{[-\eta_1 + \eta_3(\ln\frac{m_t^2}{m_c^2} + f_3(y_t))]$$
$$+ \frac{m_t^2}{m_c^2}\eta_2 f_2(y_t)\lambda^4 A^2(1 - \sigma\cos\delta)\Big\} \qquad (69)$$

Since the quantity in the curly bracket is of $0(1)$ - and so as we shall see are A and σ - one sees that in the standard model ϵ is of order

$$\epsilon \sim \lambda^4\sin\delta \quad . \qquad (70)$$

Because $\lambda^4 \sim 2\times10^{-3}$ one sees that in the standard model ϵ, and therefore CP violation in the Kaon system, is small **not** because the phase δ is particularly small, but because the interfamily mixing (represented by the factor of λ^4) is small.

Dressing of the box graph of Fig. 1 by gluons gives the QCD corrections to $|\epsilon|$ characterized by the η_i coefficients entering in Eq. (59). For ϵ', however, gluonic effects are **fundamental**, for this quantity vanishes in the limit that $\alpha_s \to 0$. The relevant diagrams that contribute to ϵ' are the, so called, Penguin diagrams of Fig. 2 [31][32]. The calculation of these diagrams is

Figure 2: Gluonic Penguin diagram contributing to ϵ'.

made simple by noting the following [33]:

Figure 3: Effective (subtracted) graph needed for Penguin computation.

i) Although each individual diagram, containing an u, c or t propagator, is divergent, the piece that is relevant for ϵ' is convergent since it is the part of the Penguin amplitude which is proportional to q^2 - the gluon momentum transfer.

ii) This q^2 factor with the $1/q^2$ factor from the gluon propagator, leads to an effective 4-Fermi interaction.

iii) The leading contribution for the Penguin diagrams is easily computed in a 4-Fermi limit for the W exchange, being simply proportional to the logarithmic divergent piece of the diagrams in this limit. To get the physical relevant answer, one then only needs to replace $\ln\Lambda^2$ by $\ln M_W^2$, with a Λ being the cutoff.

Using the above, it is straightforward to derive the effective Penguin interaction for each quark i by computing the logarithmic divergent piece of the (subtracted) graph shown in Fig. 3. One finds in this way [32]

$$\mathcal{L}_{\text{gluonicPenguin}} = \frac{G_F}{\sqrt{2}} \frac{\alpha_s}{12\pi} A_P \left\{ (\bar{s}\gamma_\mu(1-\gamma_5)\lambda_a d) \cdot \sum_q (\bar{q}\gamma^\mu \lambda_a q) \right\} \qquad (71)$$

where λ_a are $SU(3)$ matrices. The coefficient A_P in the limit in which $m_i \ll M_W$ - something we know is not true for m_t, but which will be corrected

below - is given simply by

$$A_P = \sum_i \lambda_i \ell n \ M_W^2/m_i^2 \ \ . \tag{72}$$

Using the CKM unitarity [Eq. (58)] this can be rewritten as

$$A_P = \lambda_t \ \ell n \ m_c^2/m_t^2 + \lambda_u \ \ell n \ m_c^2/m_u^2 \ \ . \tag{73}$$

For CP violating phenomena only the imaginary part of the above is relevant, and since $Im\lambda_u = 0$, one sees that what effectively dominates is the t-quark diagram in Fig. 2 with

$$Im\lambda_t = -A^2\lambda^5\sigma\sin\delta \ \ . \tag{74}$$

Because all quark species q in Eq. (71) are summed over with equal weight, it is clear that the gluonic Penguin operator carries $I = \frac{1}{2}$. Thus, it contributes only to ImA_0 in the formula [c.f. Eq. (42)] for ϵ'. One has

$$ImA_0 = C_P < \pi\pi; 0|\bar{s}\gamma^\mu(1 - \gamma_5)\lambda_a d \sum_q (\bar{q}\gamma_\mu\lambda_a q)|K^0 > \tag{75}$$

where

$$C_P = [\frac{G_F}{\sqrt{2}}\lambda] \cdot [\frac{\alpha_s}{12\pi} \ \ell n\frac{m_t^2}{m_c^2}] \cdot [A^2\lambda^4\sigma\sin\delta] \ \ . \tag{76}$$

The three terms in square brackets above characterize different physical contributions to ϵ'. $G_F\lambda/\sqrt{2}$ is the strength associated with a typical Kaon weak decay matrix element. Indeed ReA_0 has precisely this strength:

$$ReA_0 = \frac{G_F\lambda}{\sqrt{2}} < \pi\pi; 0|\bar{s}\gamma^\mu(1 - \gamma_5)u\bar{u}\gamma_\mu(1 - \gamma_5)d|K^0 > \ \ . \tag{77}$$

The factor of $\frac{\alpha_s}{12\pi} \ \ell n \ \frac{m_t^2}{m_c^2}$ reflects the fact that this contribution to ϵ' arises as a result of QCD and would vanish in the limit of degenerate quark masses. Finally the last bracket in Eq. (76) contains the same family mixing suppression factor $A^2 \ \lambda^4\sigma\sin\delta$ that enters in ϵ, which is prototypical of the CKM paradigm.

The coefficient C_P in Eq. (76) is not quite correct, since it was derived in the limit that $m_t \ll M_W$. This can be readily remedied [34]. Furthermore one needs also to incorporate higher order QCD corrections [35] into C_P.

The net result of doing both these things is to replace the second factor in Eq. (76) by a more complicated function than that given in this equation. For a typical range of m_t values ($100\ GeV < m_t < 200\ GeV$) and for a QCD scale μ appropriate to the problem at hand ($\mu \sim m_K$), this more accurate calculation gives for this factor a numerical value of about 0.1 [34]:

$$\frac{\alpha_s}{12\pi}\ \ell n\frac{m_t^2}{m_c^2} \to 0.085 \pm 0.035\ , \tag{78}$$

where the error in the above includes that produced by variations in the QCD scale and on the value of m_t.

Besides this factor, the contribution of ϵ' relative to ϵ is further reduced by the $\Delta I = 3/2$ suppression factor contained in the ratio

$$\frac{1}{\sqrt{2}}\frac{ReA_2}{ReA_0} \simeq 0.032\ \ , \tag{79}$$

where the numerical value follows from the experimental measurement of the rate for $K^+ \to \pi^+\pi^0$ relative to that of $K_S \to \pi^+\pi^-$. Hence, as an order of magnitude estimate for ϵ'/ϵ, one has

$$\epsilon'/\epsilon \sim \frac{1}{\sqrt{2}}\frac{ReA_2}{ReA_0}[\frac{\alpha_s}{12\pi}\ \ell n\frac{m_t^2}{m_c^2}]_{\rm eff}\ \simeq 3\times 10^{-3}\ \ , \tag{80}$$

which is in rough accord with the experimental result given in Eq. (4).

The above estimate ignores the fact that ϵ' and ϵ involve quite different operator matrix elements. Nevertheless, it is quite gratifying to see that in the CKM paradigm $\epsilon'/\epsilon \ll 1$ as a result of the $\Delta I = 3/2$ suppression and the fact that ϵ' vanishes as $\alpha_s \to 0$. In fact, in detail the situation regarding ϵ' is actually much more complicated. In addition to the gluonic Penguin diagrams of Fig. 2, there are electroweak Penguin diagrams - of which some examples are shown in Fig. 4 - which contribute to the $\Delta I = 3/2$ amplitude ImA_2. Although these electroweak Penguin contributions are suppressed relative to those of the gluonic Penguins by a factor of α/α_s, these contributions are not negligible since they have no $\Delta I = 3/2$ suppression. From Eq. (42) one sees that the term in ϵ' proportional to ImA_2 is not divided by ReA_0 but by ReA_2. Hence, really, electroweak Penguins are relatively enhanced. In effect, one has:

$$\frac{ImA_2}{ReA_2} = \frac{ReA_0}{ReA_2}\cdot\frac{ImA_2}{ReA_0} \simeq 20\ \frac{ImA_2}{ReA_0}\ \ . \tag{81}$$

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Figure 4: Some examples of electroweak Penguin diagrams contributing to ImA_2.

Furthermore, as shown by Flynn and Randall [36], these contributions for large m_t grow like m_t^2, not as $\ell n\ m_t^2$, and tend to cancel those of the gluonic Penguins. Thus an accurate estimate of ϵ'/ϵ in the CKM paradigm requires considerable more care. I will return to this point below.

IV.D   Comparison with Experiment

In the CKM paradigm the experimental value for ϵ can be used to determine the phase δ. Although $\epsilon \sim \sin\delta$, it is not simple to extract a precise value of δ from the precisely measured value of ϵ. This is because the relationship between ϵ and $\sin\delta$ is a function of other parameters, like A, σ, m_t, m_c and B_K which are relatively poorly known. In what follows I shall use the value of B_K given by the lattice QCD computation of this parameter [c.f. Eq. (63)] and shall take $m_c = 1.4\ GeV$, letting m_t range. For this value of m_c and $m_t < 200\ GeV$, the formula for $|\epsilon|$, Eq. (67), is well approximated by [29].[6]

$$|\epsilon| = (2.7 \pm 0.7) \times 10^{-3} A^2 \sigma \sin\delta \{1 + \frac{4}{3}(\frac{m_t}{M_W})^{1.6} A^2 (1 - \sigma\cos\delta)\} \quad . \quad (82)$$

---

[6]The error in Eq. (82) is mostly due to the uncertainty in the hadronic matrix element, typified by B_K.

Table 1: Representative Results for V_{cb} and A

Technique	$|V_{cb}|$	A
Inclusive Spectrum [40]	0.047 ± 0.004	0.97 ± 0.08
Exclusive Decay [40]	0.041 ± 0.006	0.85 ± 0.12
Heavy Quark Limit [39]	0.045 ± 0.007	0.93 ± 0.14
Heavy Quark Limit [41]	0.041 ± 0.005	0.85 ± 0.10

To proceed further one needs a value for the parameters A and σ which enter in the CKM matrix.

The parameter A is essentially fixed by the V_{cb} matrix element, while σ depends on how well one can determine the ratio of V_{ub} to V_{cb} in V_{CKM}. The values for V_{cb} obtained from studying **inclusive** semileptonic decays of B mesons to hadrons with charm $[B \rightarrow X_c \ell \nu_e]$ typically seem to be somewhat larger than those obtained by analyzing specific **exclusive** modes. In general, one expects that the former analysis be somewhat more reliable, as one can exercise some control by demanding a simulaneous fit of the lepton spectrum [37]. However, new theoretical ideas, connected with a heavy quark expansion [38] can be used to give absolute predictions for certain exclusive modes - like $B \rightarrow D^* \ell \nu_e$ at zero recoil [39]. The values of V_{cb} extracted by these means potentially should be the most accurate, once there are enough statistics for these processes. In Table 1, I display a representative set of results for V_{cb} and A obtained by these different techniques. A sensible choice, and the one I shall adopt, appears to be to take

$$|V_{cb}| = 0.043 \pm 0.005 \; ; \quad A = 0.90 \pm 0.10 \; . \tag{83}$$

To extract $|V_{ub}|/|V_{cb}|$ from experiment one studies the semileptonic decays of B mesons $(B \rightarrow X_u \ell \nu_\ell)$ in a region of momentum of the emitted lepton $(p_\ell > 2.3 \; GeV)$ which insures kinematically that the hadronic states X_u do not contain a charmed quark. That is, for $p_\ell > 2.3 \; GeV$ the data should only measure decays in which the transition $b \rightarrow u$ occurred. However, to extract a value of $|V_{ub}|$ from this analysis is non trivial, since one must be able to estimate precisely the hadronic matrix elements involved in the $B \rightarrow X_u$ transition. When one does this estimate by employing, as in the

Figure 5: Box graph which gives rise to $B_d^0 - \bar{B}_d^0$ mixing.

ACM model [37], a parton model - which is sensible in my mind, since one is summing over all states X_u - one gets a fairly large value for this matrix element and hence a rather small value for $|V_{ub}|/|V_{cb}|$. On the other hand, if one estimates the transition $B \to X_u$ by summing only over some (assumed dominant) exclusive channels, as in the ISGW model [42], the strength of the transition is smaller and, consequently, one deduces a larger value for $|V_{ub}|/|V_{cb}|$.

Using only the more recent and more accurate data obtained by CLEO II, Cassel [43] quotes the following values for $|V_{ub}|/|V_{cb}|$ extracted, respectively, using the ACM model [37] and the ISGW model [42].

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.07 \pm 0.01 \leftrightarrow \sigma = 0.32 \pm 0.06 \quad \text{ACM Model}$$
$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.11 \pm 0.02 \leftrightarrow \sigma = 0.50 \pm 0.09 \quad \text{ISGW Model} \tag{84}$$

As a preferred value, Cassel takes the average of these two results and expands somewhat the errors by including other model uncertainties [43]:

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.085 \pm 0.045 \leftrightarrow \sigma = 0.39 \pm 0.21. \tag{85}$$

It has become conventional to present the result of a CKM analysis of the data as contour plots in the $\rho - \eta$ plane, where recall $\rho = \sigma \cos \delta$ and

$\eta = \sigma \sin \delta$. The theoretical formula for $|\epsilon|$ given in Eq. (82), using the experimental value for $|\epsilon| \simeq |\eta_{+-}| = 2.27 \times 10^{-3}$ and the value of A from Eq. (83), gives the following constraint on these parameters:

$$1 = (0.96 \pm 0.33)\eta\{1 + (1.08 \pm 0.24)(\frac{m_t}{M_W})^{1.6}(1 - \rho)\} \quad . \tag{86}$$

In addition, since $\sigma = \sqrt{\rho^2 + \eta^2}$, Eq. (85) constrains ρ and η to an annular region centered at $\rho = \eta = 0$:

$$\sqrt{\rho^2 + \eta^2} = 0.39 \pm 0.21 \quad . \tag{87}$$

There is a third constraint on these parameters which comes from $B_d^0 - \bar{B}_d^0$ mixing. The amount of mixing is governed by the mass difference $(\Delta m)_{Bd}$ in this system, which in turn is fixed by the box graphs shown in Fig. 5. These graphs are totally dominated by the contribution in which the fermions in the loop are top quarks. As a result, the amount of $B_d^0 - \bar{B}_d^0$ mixing gives a measure of

$$|V_{td}|^2 = A^2\lambda^6[(1 - \rho)^2 + \eta^2] \quad . \tag{88}$$

Thus the constraint coming from the experimentally determined value of $B_d^0 - \bar{B}_d^0$ mixing is another annulus in the $\rho - \eta$ plane, this time centered at the point $\rho = 1, \eta = 0$.

One can measure the amount of $B_d^0 - \bar{B}_d^0$ mixing by determining the ratio of "wrong" to "right" sign leptons in the decay of a $|(B_d)_{\text{phys}} >$ state. Recall from our treatment in Sec. IV.A that this state was one which at $t = 0$ was a pure $|B_d >$ state but which, because of the possibility of mixing, evolved in time into a linear superposition of $|B_d >$ and $|\bar{B}_d >$ states [cf. Eq. (34)]. Physically, only $\bar{B}_d$ states decay semileptonically into **negatively** charged leptons. Thus the ratio

$$\chi_d = \frac{\Gamma\Big((B_d)_{\text{phys}} \to \ell^-\bar{\nu}_\ell X\Big)}{\Gamma\Big((B_d)_{\text{phys}} \to \ell^+\nu_\ell X\Big) + \Gamma\Big((B_d)_{\text{phys}} \to \ell^-\bar{\nu}_\ell X\Big)} \tag{89}$$

is a measure of $B_d^0 - \bar{B}_d^0$ mixing.

The quantity χ_d is readily calculated using Eq. (34). For the $B_d^0 - \bar{B}_d^0$ system there is some simplification since the width difference $\Gamma_+ - \Gamma_-$ is small both compared to the widths Γ_+ and Γ_- themselves and to the mass

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difference $\Delta m = m_+ - m_-$[44]. Thus, writing $m_+ + m_- = 2m_{B_d}$ and $\Gamma_+ + \Gamma_- = 2\Gamma_{B_d}$ and using the above approximations, Eq. (34) for the $|(B_d)_{\rm phys} >$ state simplifies to:

$$|(B_d)_{\rm phys}(t) >= e^{-im_{B_d}t}e^{-\frac{1}{2}\Gamma_{B_d}t}\Big\{\cos\frac{\Delta m_{B_d}t}{2}|B_d > +$$

$$+i\eta_{B_d}e^{i\Phi_{B_d}}\sin\frac{\Delta m_{B_d}t}{2}|\bar{B}_d >\Big\} \quad . \tag{90}$$

A simple calculation then gives for χ_d the following formula

$$\chi_d = \frac{x_d^2}{2(1+x_d^2)} \tag{91}$$

where

$$x_d = (\frac{\Delta m}{\Gamma})_{B_d} = \tau_{B_d}(\Delta m)_{B_d} \quad . \tag{92}$$

The recent compilation of Cassel [43] gives the world average value

$$\chi_d = 0.145 \pm 0.018 \pm 0.018 \tag{93}$$

or

$$x_d = 0.64 \pm 0.06 \pm 0.06 \quad . \tag{94}$$

The above value for x_d, along with a value for the B_d lifetime, can be used in conjunction with the formula for $(\Delta m)_{B_d}$, obtained by evaluating the box graph of Fig. 5, to constrain $|V_{td}|$ and hence ρ and η. An analogous calculation to the one I sketched for the Kaon system in Sec. IV.C gives [26] [29]

$$x_d = \tau_{B_d}(\Delta m)_{B_d} = \tau_{B_d}\frac{G_F^2 M_{B_d}}{6\pi^2}[B_{B_d}f_{B_d}^2\eta_B]m_t^2 f_2(y_t)|V_{td}|^2 \tag{95}$$

Here the parameters $[B_{B_d}f_{B_d}^2\eta_B]$ are the counterparts in the B_d system of $[B_k f_K^2\eta_2]$ in the Kaon system . Because the b quark is heavy, one expects that the vacuum insertion approximation should work very well, so that $B_{B_d} \simeq 1$. However, now in contrast to what happens in the Kaon system, the B_d decay constant f_{B_d} is not measured. Nevertheless, this parameter can be computed also by using lattice QCD methods. Taking into account of the short distance QCD correction factor $\eta_B \simeq 0.85$ [45], the best value for the factor $\sqrt{B_{B_d}\eta_B}\,f_{B_d}$ which one obtains from lattice QCD computations is [28]

$$\sqrt{B_{B_d}\eta_B}\,f_{B_d} = (200 \pm 35)\,\,MeV \quad . \tag{96}$$

Using the above and approximating $f_2(y_t)$ in the same way as was done in the Kaon system [c.f. Eq. (82)] [29] one has that

$$x_d = [0.44 \pm 0.15] A^2 [\eta^2 + (1-\rho)^2] (\frac{m_t}{M_W})^{1.6} \quad , \tag{97}$$

where the error is essentially that coming from Eq. (96). Using the experimental values for x_d and A this gives the third constraint in the $\rho - \eta$ plane alluded above:

$$(1.80 \pm 0.77) = [\eta^2 + (1-\rho)^2] (\frac{m_t}{M_W})^{1.6} \quad . \tag{98}$$

The constraints in the $\rho - \eta$ plane coming from $|\epsilon|$, [Eq. (86)], $|V_{ub}|/|V_{cb}|$ [Eq. (87)], and from $B_d^0 - \bar{B}_d^0$ mixing, [Eq. (98)] are displayed in Fig. 6 for two cases: $m_t = 140 \ GeV$ and $m_t = 180 \ GeV$. The cross hatched region in this figure uses instead of Eq. (87) the result of Eq. (84) for $\sigma = \sqrt{\rho^2 + \eta^2}$ obtained in the ACM model [37]. Obviously the overlap region allowed by our present theoretical and experimental knowledge of $|\epsilon|$, x_d and $|V_{ub}|/|V_{cb}|$ is crucially dependent on whether one uses Eq. (87) or the more restricture ACM result. This is illustrated in Fig. 7.

Unfortunately, even assuming η to be in its most restricted range ($\eta \simeq 0.2 - 0.3$), is not sufficient to allow for a sharp prediction for ϵ'/ϵ. This arises principally from other theoretical uncertainties incurred in estimating the hadronic matrix elements of operators which contribute to ϵ'. Nevertheless, considerable progress has been made recently in trying to tackle this question, notably by groups in Rome [46] and Munich [47] who have calculated the expectations for ϵ' at next to leading order and then tried to estimate the relevant matrix elements. Because these calculations are highly technical, I will limit myself here to give a more qualitative overview of the results obtained.

Figure 6: Allowed regions in the $\rho - \eta$ plane coming from the measurements of $|\epsilon|$, $|V_{ub}|/|V_{cb}|$ and x_d.

Figure 7: Allowed region in the $\rho - \eta$ plane for $m_t = 140 \; GeV$. The shaded band is the result obtained by relying only on the ACM model.

As I discussed earlier, the ratio ϵ'/ϵ - which is essentially the same as $Re \; \epsilon'/\epsilon$ - gets contribution from two kinds of operators: $\Delta I = 1/2$ operators and $\Delta I = 3/2$ operators. The former contributions are induced by gluonic Penguins and thus are of $0(\alpha_s)$. However, since they enter in the amplitude ImA_0, the $\Delta I = 1/2$ operators are affected by the whole $\Delta I = 1/2$ suppression factor of $ReA_2/ReA_0 \simeq 1/20$ [c.f. Eq. (42)]. On the other hand, the $\Delta I = 3/2$ contributions arise from electroweak Penguin diagrams and thus are only of $0(\alpha)$. However, ImA_2 is measured relative to ReA_2 and so, effectively, it is not suppressed by the $\Delta I = 1/2$ factor of ReA_2/ReA_0. Furthermore, these contributions grow quadratically with m_t, while those of the gluonic Penguins only depends on m_t as $\ell n \; m_t$.

The structure of the result of the calculations of ϵ'/ϵ can be written as follows [46] [47]:

$$\frac{\epsilon'}{\epsilon} = A^2 \eta \left\{ < 2\pi; I = 0| \sum_i C_i 0_i |K^0 > (1 - \Omega_I) - < 2\pi; I = 2| \sum_i \tilde{C}_i \tilde{0}_i |K^0 > \right\} \tag{99}$$

Here 0_i and $\tilde{0}_i$ are, respectively, $\Delta I = 1/2$ and $\Delta I = 3/2$ operators and their coefficients C_i and $\tilde{C}_i$ have the characteristic dependence on $\alpha_s \ell n \; m_t$ and $\alpha \; m_t^2$ alluded to above. Ω_I is a correction to the $\Delta I = 1/2$ contribution, which arises as a result of isospin violation through $\pi^0 - \eta$ mixing [48] and is

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estimated to be $\Omega_I = 0.25 \pm 0.10$. Note also in the above the characteristic CKM dependence of ϵ' - for a fixed given ϵ - on the CKM parameters $A^2\eta$. Thus, even if the hadronic matrix elements were perfectly known, present uncertainties in A and η would give about a 50% uncertainty in ϵ'/ϵ - a bit less if one could restrict η to the ACM range.

It is difficult to extract directly from the work of the Rome [46] and Munich [47] groups a value for the coefficient of $A^2\eta$, typifying the hadronic uncertainty in ϵ'/ϵ. Nevertheless, from these papers, more to get a feeling for the expectations than as a hard and fast result, I infer the following. For moderate m_t - say $m_t = 140\ GeV$ - gluonic Penguins dominate. Here the uncertainty in the matrix elements is more under control, perhaps being only of order 30%. A representative prediction for m_t in this range appears to be

$$\frac{\epsilon'}{\epsilon} = (11 \pm 4) \times 10^{-4} A^2\eta \quad (m_t = 140\ GeV) \ . \tag{100}$$

For larger m_t values ($m_t \simeq 200\ GeV$) electroweak Penguins begin to be important and they tend to cancel the contributions of the gluonic Penguins. The error in the matrix element estimation remains similar in magnitude, but the central value for the overall contribution is considerably reduced. A representative prediction for $m_t = 200\ GeV$ is, perhaps,

$$\frac{\epsilon'}{\epsilon} = (3 \pm 4) \times 10^{-4} A^2\eta \quad (m_t = 200\ GeV) \ . \tag{101}$$

If one takes the above numbers at face value, one sees that, with the present range of η allowed by the information on $|\epsilon|$, x_d and $|V_{ub}|/|V_{cb}|$, the CKM paradigm tends to favor rather small values for ϵ'/ϵ. Typically, perhaps, $\epsilon'/\epsilon \simeq 4 \times 10^{-4}$, with a theory error probably of the same order! Such small values for ϵ'/ϵ are perfectly compatible with the results obtained by the E731 collaboration [7], but are a bit difficult to reconcile with the results of NA31 [6].

IV.E Other CP Violating Processes Involving Kaons

The forthcoming round of high precision experiments at CERN and Fermilab (as well as at the Frascati Φ-factory, which is presently under construction), should measure ϵ'/ϵ to an accuracy of order $\delta(\epsilon'/\epsilon) \sim 10^{-4}$. This should be sufficient to establish that there exists indeed a $\Delta S = 1$ CP violating

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phase.[7] Even so, there is a substantial effort underway to explore other suitable processes sensitive to **direct** CP violation (i.e. $\Delta S = 1$ CP violation) in the Kaon complex.

An obvious way to establish the existence of direct CP violation is afforded by $K^{\pm}$-decays. Any asymmetry between the partial rates of a K^+ into some final state f^+ and a K^- into the state f^- would be a signal of direct CP violation, since $K^+ \leftrightarrow K^-$ mixing is forbidden by charge conservation! However, estimates of the expected magnitude of the asymmetry between the rates for charged kaons into a variety of final states are quite small and prospects for detecting direct CP violation this way are rather bleak. Another possibility, which would be nice but again is difficult experimentally, would be to try to measure the equivalent of the ϵ' parameter for K_s decays, since there is some expectation that this parameter is perhaps somewhat larger than the usual ϵ' parameters [50]. However, at the Frascati Φ factory it will not even be possible to measure η_{000} at the expected level. ($\eta_{000} \simeq \eta_{+-}$), never mind getting to the level of ϵ'_{000}!

An equally challenging possibility, but again one that is quite interesting theoretically, is connected to the observations that, if K_L were a pure CP odd eigenstate, then the process

$$K_L \rightarrow \pi^0 J^*, \tag{102}$$

where J^* is a (virtual) spin-one state, is forbidden by CP. Thus, provided they are dominated by an effective spin-one state, the processes $K_L \rightarrow \pi^0 \ell^+ \ell^-$ or $K_L \rightarrow \pi^0 \nu \bar{\nu}$ could be used to test CP.[8] Of course, as in the decay of $K_L \rightarrow \pi^+ \pi^-$ one must still separate out in these decays the direct CP effects from those coming from mixing. However, here the situation is different than in the 2π case. It turns out that for the decay $K_L \rightarrow \pi^0 \ell^+ \ell^-$ both the direct and the mixing contributions are roughly of the same order of magnitude [51], while for the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay the mixing effects are in fact totally negligible

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[7]It is perhaps worthwhile re-emphasizing the obvious here. Namely that a measurement of ϵ itself is not a proof of the CKM paradigm at all, even though qualitatively the magnitude of ϵ agrees with that of the expectations of this paradigm. In fact, as Wolfenstein [49] pointed out long ago, ϵ could be purely the result of a new CP-violating $\Delta S = 2$ superweak interaction, and have nothing to do at all with any $\Delta S = 1$ CKM phase.

[8]In the case of K_L going to charged leptons, the two-photon contribution is estimated to be small, so effectively this process is dominated by a virtual spin-one state [51]. This is clearly not a problem for the decay into neutrino pairs.

[52]! Unfortunately, both of these decays are second order weak processes and therefore the expected branching ratios are tremendously small. In the forthcoming round of experiments at Fermilab, one will get near the range of interest for the $K_L \to \pi^0 \ell^+ \ell^-$ decay but no real test of CP violation will ensue.

In Table 2, which is adapted from the recent review of Winstein and Wolfenstein [53], I summarize the expectations for various other CP violating processes in the Kaon sector, along with the accuracy one may hope to reach in forthcoming experiments. It should be clear from this Table that to test for direct CP violation in this way will require yet a further round of experiments, beyond those now planned.

V. Electric Dipole Moments

The expectation for the electric dipole moment (edm) of the neutron and the electron are that these quantities are extremely small in the CKM model. For the electron case, one needs to invoke mixing in the leptonic sector and this vanishes in the limit that the neutrinos are degenerate in mass. So the edm for the electron is truly vanishingly small in the standard paradigm. It turns out that the edm for the neutron is also very suppressed. First of all, it is easy to see that no edm at the quark level appears at one-loop, since these graphs involves either $V_{di}^* V_{di}$ or $V_{ui}^* V_{ui}$ and all phase information is lost. It turns out that the quark edm (and hence the neutron electric dipole moment) also vanishes at the two-loop level[54]. There is no simple explanation, as far as I know, for this result. Indeed individual two-loop graphs are non-vanishing, but the sum of all the graphs contributing to the edm vanishes. There is no full calculation of the edm at three-loops, but one can get an order of magnitude estimate of the effect by simply festooning the two-loop graphs with gluons. This gives [13]

$$
d_n \sim e m_d \frac{G_F \alpha \alpha_s}{\pi} \frac{m_t^2 m_s^2}{M_W^4} \lambda^6 A^2 \sigma \sin \delta \sim 10^{-33} ecm \quad . \tag{103}
$$

Somewhat larger estimates than the above have been obtained by considering, for instance, the contributions to d_n which arise from two-quark graphs in a neutron [55]. This notwithstanding, it is clear that with CKM paradigm an edm for the neutron above, say, $10^{-31} - 10^{-32}$ ecm is very unlikely. Ex-

Table 2: Expectation and Prospect for Various Kaon CP Violation Experiments

Process	CKM Expectation	Experimental Prospect
$K_s \to 3\pi^0$ $\eta_{000} = \epsilon + \epsilon'_{000}$	$\frac{\epsilon'_{000}}{\epsilon} \sim 10^{-2}$	$\delta\eta_{000} \sim 5 \times 10^{-3}$ (Φ Factory)
$K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ $\Delta\Gamma = (\Gamma^+ - \Gamma^-)/(\Gamma^+ + \Gamma^-)$ $\Delta g = (g^+ - g^-)/(g^+ + g^-)$	$\Delta\Gamma < 10^{-6}$ $\Delta g < 10^{-4}$ (Dalitz plot asymmetry)	$\delta(\Delta\Gamma) \sim 5 \times 10^{-5}$ $\delta(\Delta g) \sim 5 \times 10^{-4}$ (Φ Factory)
$K^\pm \to \pi^\pm\pi^0\gamma$ $\Delta\Gamma = (\Gamma^+ - \Gamma^-)/(\Gamma^+ + \Gamma^-)$	$\Delta\Gamma < 10^{-4} - 10^{-5}$	$\delta(\Delta\Gamma) \sim 2 \times 10^{-3}$ (Φ Factory)
$K_L \to \pi^0\ell^+\ell^-$ $B(K_L \to \pi^0\ell^+\ell^-)$	$B_{\text{direct}} \sim 10^{-11} - 10^{-13}$ (depends on m_t and V_{CKM})	$B < 7 \times 10^{-11}$ (E832 FNAL)
$K_L \to \pi^0\nu\bar\nu$ $B(K_L \to \pi^0\nu\bar\nu)$	$B \sim 10^{-11} - 10^{-12}$ (depends on m_t and V_{CKM})	$B \sim 10^{-8}$ (E832 FNAL)

perimentally, such values are a factor of $O(10^6)$ below the present bounds and are essentially unreachable. This is both good and bad news. The good news is that by continuing to probe for a non-zero edm for the neutron (or the electron) one is assured that any **positive** result will necessarily be a signal of new physics. The bad news is that there is no real assurance that any such signal will be found.

In looking for signals for new physics in connection with the neutron electric dipole moment, it is useful to examine which effective operators can give rise to an edm. There are a number of effective QCD operators which can contribute to the electric dipole moment for the neutron. The most famous of these is the CP odd two-gluon operator

$$\mathcal{L}_{CP\,viol} = \bar{\theta}\frac{\alpha_s}{8\pi}G_a^{\mu\nu}\tilde{G}_{a\mu\nu} \ . \tag{104}$$

This operator arises naturally in QCD as a result of the non-trivial nature of the QCD vacuum[56] and $\bar{\theta}$ represents a combination of the vacuum angle contribution and that from the quark mass matrix ($\bar{\theta} = \theta + \text{Arg } det\ M$). The electric dipole moment for the neutron coming from such a term is enormous, unless $\bar{\theta}$ is very small. One can estimate[57] for d_n a value:

$$d_n \sim \bar{\theta}\left(\frac{m_d}{M_n^2}\right)ecm \ \sim 4 \times 10^{-16}\bar{\theta}\ ecm \ . \tag{105}$$

To agree with the experimental bound given in Eq. (10), $\bar{\theta}$ has to be vanishingly small ($\bar{\theta} \leq 10^{-10}$)! Why this should be so, is the strong CP problem. The only sensible solution, to my mind, of this conundrum is that actually $\bar{\theta} \equiv 0$ for dynamical reasons [58]. In this case, of course, the operator in Eq. (104) does not contribute to d_n.

Even if $\bar{\theta} = 0$ dynamically, one can always get a contribution to the electric dipole moment of the neutron from an induced CP odd three-gluon operator

$$\mathcal{L}_{CP\,viol} = \frac{1}{\Lambda^2}f_{abc}G_a^{\mu\nu}G_{b\nu}^{\alpha}\tilde{G}_{c\alpha\mu} \ . \tag{106}$$

The scale Λ for the CKM model is effectively extremely large because the GIM mechanism [25] introduces high powers of $(m_q/M_W)^n$. This is not necessarily so in other models and one can get a sizable edm from the operator in Eq. (106). Typically

$$d_n \sim \frac{eM_n}{\pi\Lambda^2} \tag{107}$$

and edm's in the neighborhood of 10^{-26} ecm are perfectly plausible in a variety of models[59]. For these reasons, it seems very sensible to continue to probe experimentally as hard as one can for a non-vanishing edm.

VI. CP Violation in the B System

In decays of B mesons CP asymmetries do not have to suffer from the family mixing suppression factors one encounters in the Kaon sector. After all, B_d (or B_s) mesons contain already quarks from the third and first (or second) generation and their decay by-products can easily involve states containing quarks from yet another generation. The presence of all three generations in the relevant decay amplitudes serves to remove, for certain processes, the family mixing suppression factors which arose through virtual intermediate states in the Kaon system. As we shall see, the best place to see directly the CP violating phase of the CKM paradigm is by studying CP violating asymmetries in B decays to **CP-self conjugate states** f. These are states which have the property that $\bar{f} = \pm f$. Before showing why this is so, however, it is necessary to develop a bit of formalism and detail a certain amount of information related to B decays.

Because B mesons are quite heavy, they decay into a large number of distinct channels. Thus, as we discussed in Sec. IV.D, in contrast to what happens in the Kaon system, one expects that in the neutral B sector $\Gamma_+ \simeq \Gamma_-$. Hence

$$\Gamma = \frac{1}{2}(\Gamma_+ + \Gamma_-) \gg \frac{1}{2}(\Gamma_+ - \Gamma_-) \quad . \tag{108}$$

Furthermore, for both the B_d^0 and B_s^0 systems the mass difference between the eigenstates Δm is much less than the average mass:

$$\frac{1}{2}(m_+ + m_-) \gg \frac{1}{2}\Delta m = \frac{1}{2}(m_+ - m_-) \quad . \tag{109}$$

However, as I remarked upon earlier, experimentally Δm is quite comparable to Γ for the case of the B_d states [43] [cf Eq. (94)]

$$\left(\frac{\Delta m}{\Gamma}\right)_{B_d} \equiv x_d = 0.64 \pm 0.08 \tag{110}$$

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Although there is no measurement to date of $\Delta m/\Gamma$ for the B_s system, one expects that also this quantity be of $0(1)$. As a result, the time evolution of a $|(B_s)\text{phys} >$ state will be also given by an equation similar to Eq. (90) with $B_d \leftrightarrow B_s$. In what follows, therefore, I shall not distinguish explicitly between the B_d and B_s cases - until that is needed - and write simply

$$|B_{\text{phys}}(t) > = \ e^{-im_B t} e^{-\frac{1}{2}\Gamma_B t} \left\{ \cos \frac{\Delta m_B t}{2} |B > + \right.$$

$$\left. + i\eta_B e^{i\Phi_B} \sin \frac{\Delta m_B t}{2} |\bar{B} > \right\} \ . \tag{111}$$

$$|\bar{B}_{\text{phys}}(t) > = e^{-im_B t} e^{-\frac{1}{2}\Gamma_B t} \ \left\{ \frac{i\bar{e}^{-i\Phi_B}}{\eta_B} \sin \frac{\Delta m_B t}{2} |B > \right.$$

$$\left. + \cos \frac{\Delta m_B t}{2} |\bar{B} > \right\} \ . \tag{112}$$

For the B system, the off diagonal matrix element $M_{12} \sim m_t^2$, while the off-diagonal width $\Gamma_{12} \sim m_c^2$ (since the decays $b \to c$ dominate) [44]. Thus using Eq. (33) one expects that

$$\eta_B e^{i\Phi_B} = \frac{1 - \epsilon_B}{1 + \epsilon_B} = \left[\frac{(M_{12}^* - \frac{1}{2}\Gamma_{12}^*)^{1/2}}{(M_{12} - \frac{i}{2}\Gamma_{12})^{1/2}} \right]_B \simeq \left[\frac{M_{12}^*}{M_{12}} \right]^{1/2} \equiv e^{-i\Phi_M} \ . \tag{113}$$

That is, $\eta_B \simeq 1$, with the phase Φ_B being essentially the negative of the phase Φ_M of the appropriate mixing matrix M_{12}. For the B system this mixing matrix is dominantly given by computing a box graph with a t quark in the loop [cf Fig. 5 for the B_d case]. It is easy to see that for $q = \{d, s\}$

$$(M_{12})_{B_q} \sim [V_{tb} V_{tq}^*]^2 \sim [V_{tq}^*]^2 \ , \tag{114}$$

since $V_{tb} \simeq 1$ to leading order in λ. Using Eq. (26) for the CKM matrix one observes that, to leading order in λ, V_{ts} is real while V_{td} has a complex phase. Writing

$$V_{td} = |V_{td}| e^{-i\beta} \ , \tag{115}$$

one secures the result

$$(M_{12})_{B_q} = |M_{12}|_{B_q} \cdot \begin{cases} e^{+2i\beta} & (q = d) \\ 1 & (q = s) \end{cases} \tag{116}$$

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Hence, in the CKM paradigm, for the neutral B states one can take in Eq. (113)

$$\eta_B \simeq 1 \quad , \quad \Phi_B \simeq \left\{ \begin{array}{ll} -2\beta & (q = d) \\ 0 & (q = s) \end{array} \right. \tag{117}$$

VI.A   CP Violation in Decays to CP-Self Conjugate States

It is interesting to study the decays of B_{phys} states into CP-self conjugate states f - with $\bar{f} = \pm f$. The ratio of the decay amplitudes of B and $\bar{B}$ to one of these states f would have unit magnitude if CP were conserved, but in general one expects that

$$\frac{A(\bar{B} \to f)}{A(B \to f)} = \eta_f e^{i\Phi_D} \quad . \tag{118}$$

Using this equation, it is straightforward to compute the (time dependent) rate of $B_{\text{phys}}(t)$ and $\bar{B}_{\text{phys}}(t)$ to decay into a CP self conjugate state f:

$$\Gamma(B_{\text{phys}}(t) \to f) = \Gamma(B \to f) \ e^{-\Gamma_B t} \Bigg\{ \cos^2 \tfrac{\Delta m_B t}{2} + \eta_f^2 \sin^2 \tfrac{\Delta m_B t}{2}$$
$$-\eta_f \sin(\Phi_B + \Phi_D) \sin \Delta m_B t \Bigg\} \tag{119}$$

$$\Gamma(\bar{B}_{\text{phys}}(t) \to f) = \Gamma(B \to f) \ e^{-\Gamma_B t} \Bigg\{ \eta_f^2 \cos^2 \tfrac{\Delta m_B t}{2} + \sin^2 \tfrac{\Delta m_B t}{2}$$
$$+\eta_f \sin(\Phi_B + \Phi_D) \sin \Delta m_B t \Bigg\} \quad . \tag{120}$$

From CPT conservation one can obtain a relation between the amplitudes of $A(B \to f)$ and $A(\bar{B} \to \bar{f})$. Since for CP self conjugate states $A(\bar{B} \to \bar{f}) = \pm A(\bar{B} \to f)$, this relation has a bearing on the desired ratio of Eq. (118). In general the amplitude $A(B \to f)$ will contain different weak amplitudes a_i each multiplied by an appropriate strong rescattering phase factor $e^{i\delta_i}$:

$$A(B \to f) = \sum_i a_i e^{i\delta_i} \quad . \tag{121}$$

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CPT conservation gives for the corresponding amplitude for $A(\bar{B} \to \bar{f})$

$$A(\bar{B} \to \bar{f}) = \sum_i a_i^* e^{i\delta_i} \quad . \tag{122}$$

That is, one conjugates the weak amplitudes but keeps the rescattering phases the same [cf. Eqs. (38) and (39) for the $K^0 - \bar{K}^0$ complex]. Whence it follows that

$$\eta_f e^{i\Phi_D} = \pm \frac{\sum_i a_i^* e^{i\delta_i}}{\sum_i a_i e^{i\delta_i}} \quad . \tag{123}$$

There are many circumstances where one can argue dynamically that **only one** weak amplitude dominates the ratio of the $\bar{B} \to f$ to $B \to f$ amplitudes. In this case clearly

$$\eta_f = \pm 1 \quad (\bar{f} = \pm f) \quad . \tag{124}$$

and the formulas for the decays of B_{phys} and $\bar{B}_{\text{phys}}$ simplify considerably. In particular, in this case, the asymmetry between these rates is simply a measure of the CP violating phase $\Phi_B + \Phi_D$:

$$A(t) = \frac{\Gamma(B_{\text{phys}}(t) \to f) - \Gamma(\bar{B}_{\text{phys}}(t) \to f)}{\Gamma(B_{\text{phys}}(t) \to f) + \Gamma(\bar{B}_{\text{phys}}(t) \to f)} = (\mp) \sin(\Phi_B + \Phi_D) \sin \Delta m_B t \quad . \tag{125}$$

In contrast to the Kaon case, however, this asymmetry is not small since the CP violating phases Φ_B and Φ_D are not suppressed by small mixing angles, Indeed, we just saw above that for the B_d case $\Phi_{B_d} = -2\beta$, with $-\beta$ being the phase of V_{td}.

The above nice formula holds for decays $B \to f$ and $\bar{B} \to f$ which are dominated by one weak amplitude. In this case the strong rescattering phases cancel, since they are common for the single $B \to f$ and $\bar{B} \to f$ transition amplitudes. Thus Φ_D is purely a weak CP violating phase. Because the B's are heavy, we can to a good approximation compute their decays using the spectator picture [60], in which the weak amplitude for a B (or a $\bar{B}$) to decay is just proportional to the corresponding weak amplitude for its constituent $\bar{b}$ (or b) quark to decay. That is

$$\frac{A(\bar{B} \to f)}{A(B \to f)} \simeq (\pm) \frac{A(b \to qq'\bar{q}'')}{A(\bar{b} \to \bar{q}\bar{q}'q'')} \tag{126}$$

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If one ignores Penguin effects, in the above $q = \{u, c\}$ while q' and q'' are quarks of the first two generations with $q' = \{d, s\}$ and $q'' = \{u, c\}$. If it were not for the presence of the CKM phase, the b quark decays amplitudes would be real. Since according to Eq. (26), to leading order in λ, $V_{q''q'}$ is real, one sees that in this approximation:

$$\frac{A(\bar{B} \to f)}{A(B \to f)} \simeq (\pm)\frac{V_{qb}}{V_{qb}^*} = (\pm)\left\{ \begin{array}{ll} e^{-2i\delta} & (b \to u \text{ transition}) \\ 1 & (b \to c \text{ transition}) \end{array} \right. . \tag{127}$$

Thus the decay phase Φ_D either is directly related to the CKM phase δ or vanishes! That is,

$$\Phi_D \simeq \left\{ \begin{array}{ll} -2\delta & (b \to u \text{ transition}) \\ 0 & (b \to c \text{ transition}) \end{array} \right. \tag{128}$$

VI.B The Unitarity Triangle and Classes of Predictions

When one weak amplitude dominates in the decay of neutral B mesons to CP self-conjugate states, the decays of $B_{\text{phys}}(t)$ and $\bar{B}_{\text{phys}}(t)$ into these states only differ by the sign of the modulating factor $\sin(\Phi_B + \Phi_D)\sin \Delta m_B t$. As a result, the asymmetry between these rates, normalized to the sum of the rates, measures precisely this factor [c.f. Eq. (125)]. The coefficient of $\sin \Delta m_B t$ is a measure of CP violation in the $B - \bar{B}$ complex. Here, however, in contrast to what obtains in the Kaon case, one expects this coefficient to be sizable.

Let us write:
$$\alpha_f = \mp \sin(\Phi_B + \Phi_D) . \tag{129}$$

As we saw in the last section, in the CKM model these CP violating phases are directly related to the phases of the complex CKM matrix elements. Furthermore, to leading order in λ, we identified only four possibilities for these phases, which are described by Eqs. (117) and (128).[9] Thus one can divide the expectations for the coefficients α_f into four different classes, depending on whether one is dealing with B_d or B_s decays and depending

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[9]Φ_B and Φ_D are convention dependent and the results given are those which follow if one uses the quark phase convention. However, the sum $\Phi_B + \Phi_D$ is convention independent, as it must be.

Table 3: CKM Model expectations for the hadronic asymmetry coefficients α_f to CP-self conjugage states $f = \pm \bar{f}$

Process	α_f	Typical example
B_d decay ; $b \to c$ transition	$\pm \sin 2\beta$	$B_d \to \psi K_s$
B_d decay ; $b \to u$ transition	$\pm \sin 2(\beta + \delta)$	$B_d \to \pi^+ \pi^-$
B_s decay ; $b \to c$ transition	0	$B_s \to \psi \phi$
B_s decay ; $b \to u$ transition	$\pm \sin 2\delta$	$B_s \to \pi^0 K_s$

on whether the main weak transition involves a $b \to c$ or a $b \to u$ process. The relevant results are displayed in Table 3, in which also typical decay processes are identified.

There is a very nice interpretation of the above results, whose origins can be traced to Bjorken[61], although the realization that the CKM model has the simple set of results of Table 2 probably predates this interpretation [62]. The angles which enter the different asymmetries in Table 3 $\{\delta, \beta, \delta + \beta\}$ are angles of a triangle intimately connected with the CKM matrix. Indeed, the existence of this (approximate) triangle is just a simple reflection of the unitary of V_{CKM}. Consider for this purpose the unitarity relation for the bd matrix element of $V_{CKM}^\dagger V_{CKM}$. Since V_{CKM} is unitary one has

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \quad . \tag{130}$$

The above, using Eq. (26), reduces to leading order in λ to

$$V_{ub}^* + V_{td} \simeq \lambda V_{cb}^* \tag{131}$$

or

$$|V_{ub}|e^{i\delta} + |V_{td}|e^{-i\beta} \simeq A\lambda^3 \quad . \tag{132}$$

The above describes a triangle in the complex plane, two of whose angles are δ and β, as shown in Fig. 8a. Using that

$$\begin{aligned}
V_{ub} &= & A\lambda^3 \sigma e^{-i\delta} &= & A\lambda^3(\rho - i\eta) \\
V_{td} &= & A\lambda^3(1 - \sigma e^{i\delta}) &= & A\lambda^3(1 - \rho - i\eta) \quad ,
\end{aligned} \tag{133}$$

Figure 8: a) Unitarity triangle showing the angles β and δ; b) This same triangle rescaled, shown in the $\rho - \eta$ plane.

and scaling Eq. (132) by $A\lambda^3$, one sees that the "unitarity" triangle of Fig. 8a becomes the triangle in the $\rho - \eta$ plane shown in Fig. 8b.

As can be seen from Fig. 8b, the tip of the unitarity triangle is the point $\{\rho, \eta\}$. As we discussed in Sec. IV.D, this point is **not** uniquely determined by the present measurements of Kaon CP violation, V_{ub}/V_{cb} and $B - \bar{B}$ mixing, mostly due to uncertainties in the theory. From our analysis of these measurements, the allowed region in the $\eta - \rho$ plane is that shown in Fig. 7. Even restricting oneselves to the small strip favored by the ACM model, one still has a variety of possible unitarity triangles and two of these are shown in Fig. 9. Even given this uncertainty, in all cases, it appears that the important angles δ and β, and their complement $(\delta + \beta)$, which determine α_f are sizable. Thus in the CKM model one expects CP violating asymmetries α_f which are at the level of 10% or so, rather than at the level of 10^{-3} which is what is observed in the neutral Kaon system.

There are a number of analyses in the literature for the expectations of the CKM model for the asymmetry coefficients α_f [63]. I give in Fig. 10, as an example, the results of Nir [13] in which the allowed region in

Figure 9: Examples of allowed unitarity triangles which follow from our analysis of the Cabibbo Kobayashi Maskawa matrix.

Figure 10: Allowed values in the $\sin 2\beta - \sin 2(\beta + \delta)$ plane, according to the analysis of Nir [13].

the $\sin 2\beta - \sin 2(\beta + \delta)$ plane is shown. As can be seen from this figure, there is always a sizable asymmetry coefficient $(\sin 2\beta)$ for B_d decays to CP self-conjugate states involving a $b \to c$ transition. Thus the decays $B_d \to \psi K_s$, $\bar{B}_d \to \psi K_s$, prototypical of these processes, should be prime candidates for observing CP violation in the neutral B system.

The above nice results are predicated on having only one weak amplitude dominate the decays in question. It is clearly important to examine the limits of validity of this approximation. This is particularly necessary for decays into CP self-conjugate states, since in these cases the quark decays $b \to qq'\bar{q}''$ always have $q'' = q$. Thus for these decays, associated with the quark decay amplitude there **always** exists also a Penguin amplitude $b \to q'(q\bar{q})$. This is shown, schematically, in Fig. 11.

Let us examine what corrections the Penguin contributions of Fig. 11 give to the asymmetry coefficients α_f. Because one now does not have a single weak amplitude, the simple relation (124) no longer obtains and one must use the general formula (123) for the ratio of the decay amplitudes $A(\bar{B} \to f)$ to $A(B \to f)$. In particular, if we denote the quark decay amplitude by A_q

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Figure 11: Decay and Penguin amplitudes entering in the processes $b \to q'(q\bar{q})$.

and the Penguin amplitude by A_P, one has

$$\frac{A(\bar{B} \to f)}{A(B \to f)} = \pm \frac{A_q^* e^{i\delta_q} + A_P^* e^{i\delta_P}}{A_q e^{i\delta_q} + A_P e^{i\delta_P}} \quad , \tag{134}$$

where δ_q and δ_P are the rescattering phases corresponding to the quark decay and the Penguin amplitudes, respectively. In general, these strong phases now no longer cancel and the measured asymmetry is no longer simply related to the unitarity triangle phases.

Fortunately, a careful analysis by various people [64] has shown that, in most instances, the Penguin contributions are quite negligible. For instance, for the important process $B_d \to \psi K_S$ the relevant Penguin graph involves a $b \to s$ Penguin. This graph, like the associated quark decay amplitude which involves a $b \to c$ transition, is real to leading order in λ. Hence, in this case both A_q and A_P are real and the ratio of the amplitudes in Eq. (134)–even taking into account the Penguin effects–is again approximately ± 1. Typical estimates of the uncertainty in α_f due to the presence of Penguin graphs are given in [64] and are of the order of $\delta\alpha_f/\alpha_f \leq 1\%$ for the process $B_d \to \psi K_S$

and are of order of $\delta\alpha_f/\alpha_f \leq 10\%$ for the processes $B_d \rightarrow \pi^+\pi^-$.

VI.C Other CP Violating Asymmetries in the Neutral B Systems

The holy grail for testing the CKM model is the measurement of the CP violating asymmetries of neutral B decays to CP self-conjugate states. As we just discussed, these asymmetries afford a direct way to check the unitarity of the mixing matrix through the determination of the angles in the unitarity triangles. However, even though one expects in general rather large asymmetries, these measurements are far from trivial experimentally. What makes them difficult is that the branching ratios for the various exclusive decays $B \rightarrow f$ one wants to study are small and, furthermore, to obtain a value for α_f one needs to know how the initial state was "born". The necessity to tag the events for these purposes, along with the small exclusive branching ratios, require samples of the order of $10^7 - 10^8$ B's for a good measurement. Such samples will become available in the future in the e^+e^- B factories now projected in the USA and Japan. Samples even larger than these are already being produced yearly at the Tevatron Collider. Here, however, the challenge is to extract the signal from the background.

Given these practical difficulties, it is worthwhile contemplating whether there are other clean tests of the CKM paradigm, besides those afforded by studying B decays to CP self-conjugate states. It turns out that, provided one can do a certain amount of **spin tagging**, there are a number of other measurements in the neutral B system which are as theoretically pristine as those we discussed in the previous section. Some of the ideas behind these alternate tests were first discussed in a paper by Kayser *et al.* [65] and I will use two of the examples considered in this paper to illustrate the general principles. More detailed discussions can be found in [66].

A nice illustration of the complications which spin introduces when discussing CP violating asymmetries is provided by the decay $B_d \rightarrow p\bar{p}$. Even though this final state is a CP self-conjugate state, it is not quite correct that the CP-violating asymmetry of $(B_d)_{phys}$ to $p\bar{p}$ provides a clean test of the CKM model. This is because the $p\bar{p}$ state, in contrast to the examples considered earlier, has two possible helicities and not just one. Since CP flips helicity, not having a pure helicity final state necessitates knowing something

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of the hadronic dynamics to extract the desired asymmetry coefficient. The CP eigenstates for the $p\bar{p}$ system are linear combinations of the two states which are produced in B_d decay, which either have both p and $\bar{p}$ carrying helicity $+1/2$, or both carrying helicity $-1/2$. That is, the CP eigenstates are

$$|p\bar{p};\ \pm> = \frac{1}{\sqrt{2}}\left\{|p+\frac{1}{2}>|\bar{p}+\frac{1}{2}> \pm |p-\frac{1}{2}>|\bar{p}-\frac{1}{2}>\right\}\ . \qquad (135)$$

The decay rates of $(B_d)_{phys}$ to the above states will have a modulating factor whose constant of proportionality $\alpha_{p\bar{p}}$ is entirely fixed by the CKM matrix:[10]

$$\Gamma((B_d)_{\text{phys}}(t) \to p\bar{p};\pm) = |A_\pm|^2 e^{-\Gamma_B t}\{1 \pm \alpha_{p\bar{p}}\sin\Delta m_B t\}. \qquad (136)$$

Because, in general, $|A_+|^2 \neq |A_-|^2$ the rate of $(B_d)_{\text{phys}}$ to $p\bar{p}$, however, depends on the dynamics. One has

$$
\begin{aligned}
\Gamma((B_d)_{\text{phys}}(t) \to p\bar{p}) &= \Gamma((B_d)_{\text{phys}}(t) \to p\bar{p};+) \\
&\quad + \Gamma((B_d)_{\text{phys}}(t) \to p\bar{p};-) \\
&= (|A_+|^2 + |A_-|^2)e^{-\Gamma_B t} \\
&\quad \times\left\{1 + \left(\frac{|A_-|^2 - |A_+|^2}{|A_-|^2 + |A_+|^2}\right)\alpha_{p\bar{p}}\sin\Delta m_B\, t\right\}
\end{aligned}
\qquad (137)
$$

Thus, unless one can determine the rates $|A_+|^2$ and $|A_-|^2$ independently, one cannot extract $\alpha_{p\bar{p}}$ from a measurement of the asymmetry between the decays of B_d and $\bar{B}_d$ to $p\bar{p}$.

The experimental selection of a pure CP eigenstate for the $p\bar{p}$ system is not practical [65]. However, this selection can be done quite readily in other instances. A nice example is provided by the decay of B_d into ψK^*. As with the $p\bar{p}$ case, here the final state is not a pure helicity eigenstate, since the ψ and the K^* can both have either $\lambda = +1$, $\lambda = 0$ or $\lambda = -1$. However, one can isolate the final state where the ψ and the K^* have helicity $\lambda = 0$, by studying the angular distributions of the subsequent decay of the produced

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[10]Indeed, $\alpha_{p\bar{p}} = -\alpha_{\pi^+\pi^-}$.

K^* into $K_S\pi^0$. If θ is the angle between the K_S direction and that of the K^*, in the K^* rest frame, then one can show that [65]

$$\frac{d\Gamma}{d\cos\theta}[(B_d)_{\text{phys}}(t) \to K^*\psi \to K_S(\theta)\pi^0\psi] = \Gamma_0(t) + \Gamma_1(t)\sin^2\theta. \quad (138)$$

The rate $\Gamma_1(t)$ has a modulating factor whose coefficient is not purely determined by the weak interactions. However, $\Gamma_0(t)$ is precisely $\Gamma((B_d)_{\text{phys}}(t) \to K^*\psi; \ \lambda = 0)$ and as such it provides an alternative measurement of the asymmetry coefficient $\alpha_{K_S\psi}$. One has

$$\Gamma_0(t) = \Gamma((B_d)_{\text{phys}}(t) \to K^*\psi; \ \lambda = 0) = \text{const.}\ e^{-\Gamma_B t}\{1 - \alpha_{K_S\psi}\sin\Delta m_B\ t\}. \quad (139)$$

That is, this rate has the same coefficient for the modulating factor as that entering in the decay of $B_d \to \psi K_S$, except that the sign is opposite since the K^* has spin 1 while the K_S has spin 0.

VI.D   CP Violating Asymmetries in Charged B Decays

The decays of charged B mesons, like those of charged Kaons, can be used to look for CP violation effects. To observe a CP violation in $B^\pm$ decays one needs to have an interference between two amplitudes[11]. Furthermore these amplitudes must have both a **weak** and a **strong** relative phase between each other to lead to an observable effect. In view of this, let us write for the rate of a B^+ to decay to some final state f^+ the expression

$$\Gamma(B^+ \to f^+) = |A_1 + A_2 e^{i\delta_W} e^{i\delta_S}|^2, \quad (140)$$

where δ_W, δ_S, are, respectively, the weak and strong phase differences between the amplitudes A_1 and A_2, which otherwise are taken to be real. It follows then that the rate of B^- decay to f^- is given by

$$\Gamma(B^- \to f^-) = |A_1 + A_2 e^{-i\delta_W} e^{i\delta_S}|^2. \quad (141)$$

---

[11]For the decay of neutral B's to CP self-conjugate states one also has, in effect, two amplitudes–the actual decay amplitude of the B mesons themselves and the amplitude for $B - \bar{B}$ mixing.

That is, the weak phase enters with the opposite sign as a result of charge conjugation, but one retains the same final state strong rescattering phase. From these formulas it follows, as advertised, that the asymmetry between these rates–which is a measure of CP violation–vanishes unless both $\delta_W \neq 0$ and $\delta_S \neq 0$ [12]:

$$A_{+-} = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)} = \frac{2A_1 A_2 \sin \delta_W \sin \delta_S}{A_1^2 + A_2^2 + 2A_1 A_2 \cos \delta_W \cos \delta_S}. \tag{142}$$

Because A_{+-} specifically depends on the strong relative phase δ_S, estimates of these asymmetries always involve strong dynamics and thus are more uncertain. Furthermore, as one can see from the above formula, to get a sizable asymmetry one must interfere amplitudes which are roughly of the same order of magnitude, since if $A_1 \ll A_2$ then $A_{+-} \sim O(A_1/A_2)$. Interestingly enough, there are certain types of $B^\pm$ decays where this circumstance obtains. Unfortunately, this typically happens for decays which have rather small branching ratios and so the observation of a possible rate asymmetry A_{+-} will be difficult due to lack of statistics.

A good illustration of the above considerations is afforded by the doubly CKM suppressed decays involving, at the quark level, the transition $b \to u\bar{u}s$. Any of these decays can proceed either via a direct quark decay, via W exchange, or via a $b - s$ Penguin graph as illustrated in Fig. 12. The direct decay graph is doubly suppressed because it involves a $b \to u$ vertex as well as a $u \to s$ vertex. Hence

$$A_{direct\ decay} \sim O(\lambda^4). \tag{143}$$

The $b-s$ Penguin graph, since it is dominated by t and c intermediate states is only of $O(\lambda^2)$, but of course it is suppressed below this level because of the presence of the gluon. This suppression can make it effectively to be of $O(\lambda^4)$ also

$$A_{Penguin} \sim [\text{Penguin suppr.}]O(\lambda^2) \sim O(\lambda^4). \tag{144}$$

Since only V_{ub} has a nontrivial phase and the u quark contribution in the Penguin graph is totally negligible,, one sees that the two amplitudes in Fig.

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[12] One needs to have, of course, also both A_1 and A_2 be nonvanishing.

Figure 12: Graphs contributing to the process $b \rightarrow u\bar{u}s$: (a) direct decay graph; (b)$b - s$ Penguin graph.

12 have a relative weak phase between them. Indeed, in this case $\delta_W \simeq \delta$, the CKM phase itself. For any given final state, say $B^+ \rightarrow K^+\rho^0$, one also expects that the Penguin amplitude and the direct decay amplitude give rise to different rescattering phases. In fact, one can twist the Penguin graph involving the c quark intermediate state so that it can mimic a $D_s\bar{D}$ state.[67] So naively one might expect that the Penguin amplitude should carry the strong phase associated with the rescattering process $D_s\bar{D} \rightarrow K^+\rho^0$, while the decay amplitude would not have such a rescattering phase.

Because of the difficulties of computing the strong rescattering phases, the estimates for the asymmetry A_{+-} expected for the decays $B^{\pm} \rightarrow K^{\pm}\rho^o$ found in the literature vary considerably, from less than a percent to over 10% [68]. Because this process is suppressed by higher powers of CKM mixing angles, one expects branching ratios for these decays below 10^{-5}, making the detection of even a substantial asymmetry very difficult. Thus the prospects of finding CP violation in charged B decays are not very promising. Asymmetries in the rates of charged B decay should certainly be looked for, because they are in many ways simpler experimentally (no need to tag, mostly charged tracks in the final state, etc.). However, their interpretation necessarily will need strong interaction input and one will have to be very lucky to see a signal at all!

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